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High temperature expansion in supersymmetric matrix quantum mechanics

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Abstract: We formulate the high temperature expansion in supersymmetric matrix quantum mechanics with 4, 8 and 16 supercharges. The models can be obtained by dimensionally reducing $\mathcal{N} = 1$ U(N) super Yang-Mills theory in $D = 4, 6, 10$ to 1 dimension, respectively. While the non-zero frequency modes become weakly coupled at high temperature, the zero modes remain strongly coupled. We find, however, that the integration over the zero modes that remains after integrating out all the non-zero modes perturbatively, reduces to the evaluation of connected Green's functions in the bosonic IKKT model. We perform Monte Carlo simulation to compute these Green's functions, which are then used to obtain the coefficients of the high temperature expansion for various quantities up to the next-leading order. Our results nicely reproduce the asymptotic behaviors of the recent simulation results at finite temperature. In particular, the fermionic matrices, which decouple at the leading order, give rise to substantial effects at the next-leading order, reflecting finite temperature behaviors qualitatively different from the corresponding models without fermions.

Keywords: Matrix Models, Thermal Field Theory.

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Contents

1. Introduction

Recently large-N gauge theories are playing increasingly important roles in string theory. One of the crucial discoveries was that U(N) gauge theory appears as a low energy effective theory [1] of a stack of N D-branes [2] in string theory. This led to various interesting conjectures. For instance, large-N gauge theories obtained by dimensionally reducing 10d U(N) super Yang-Mills theory to $0, 1, 2$ dimensions are conjectured to provide non-perturbative formulations of superstring/M theories [3 − 5].

Another type of conjectures asserts the duality between strongly coupled large-N gauge theory and weakly coupled supergravity. In the AdS/CFT correspondence [6], for instance, it is conjectured that 4d $\mathcal{N} = 4$ U(N) super Yang-Mills theory is dual to the type IIB supergravity on $\mathrm{AdS}_5 \times \mathrm{S}^5$. This duality is generalized to the finite temperature setup [7] and to non-conformal gauge theories [8]. Motivated by such dualities, large-N gauge theories at finite temperature [9 − 12] have been studied intensively.

Monte Carlo simulation is expected to be a powerful approach to explore the phase diagram of large-N gauge theories. Indeed there was a remarkable progress in this direction recently. Supersymmetric matrix quantum mechanics have been studied by Monte Carlo simulation for the first time [13 − 15]. Ref. [15], in particular, deals with the model with 16 supercharges, which may be viewed[1] as the low energy effective theory of N D0-branes in the type IIA superstring theory [1]. The Monte Carlo results confirmed the gauge/gravity duality from first principles. Unlike in the bosonic model [16, 10, 17], no

[1]The model is formally the same as the non-perturbative formulation [3] of the M theory although the large-N limit should be taken in a different way.

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phase transition was observed at finite temperature, which is consistent with the prediction based on the gauge/gravity duality [18, 11]. Moreover, the internal energy at low temperature agreed with that of the non-extremal black hole described by the dual geometry. This implies in particular that the Bekenstein-Hawking entropy of the black hole is given a microscopic origin in terms of the open strings attached to the constituent N D0-branes. Unlike in Strominger-Vafa's result [19] for the extremal black hole, which relied on the supersymmetric non-renormalization theorem, the agreement has been found by studying the strongly coupled dynamics of the D0-brane effective theory directly. See refs. [20, 8] for earlier works, which connect the supersymmetric matrix quantum mechanics at finite temperature to the black-hole physics through the gauge/gravity duality.

In this paper we formulate the high temperature expansion in the supersymmetric matrix quantum mechanics. While the low temperature behavior of the theory describes the classical black hole, the high temperature behavior is expected to describe hot strings [21]. We study the models with $4, 8, 16$ supercharges that can be obtained by dimensionally reducing $\mathcal{N} = 1$ U(N) super Yang-Mills theory in $D = 4, 6, 10$ to 1 dimension. The high temperature limit of the $D = 10$ case [3] for $N = 2$ has been studied in ref. [22]. As observed there and also in refs. [10, 23], only the bosonic zero modes survive at the leading order, and their dynamics are governed by the bosonic part of the IKKT [4] matrix model. In order to see the effects of the fermions, we proceed to the next-leading order. After integrating out the weakly-coupled non-zero frequency modes perturbatively, we find that the remaining integration over the zero modes reduces to the evaluation of connected Green's functions in the bosonic IKKT model. This can be done by Monte Carlo simulation with much less effort than simulating the supersymmetric models at finite temperature directly. In particular, we are able to make a reliable large-N extrapolation using the data for N up to 32. We calculate the internal energy, the Polyakov line, and the extent of the eigenvalue distribution explicitly for $D = 4, 6, 10$. Our results nicely reproduce the asymptotic behaviors of the recent Monte Carlo data obtained for both supersymmetric models and bosonic models at finite temperature. The different properties of the two classes of models are clearly reflected in the next-leading order terms.

The rest of this paper is organized as follows. In section 2 we define the model and the observables we study in this paper. In sections 3 and 4 we present the calculations at the leading order and at the next-leading order, respectively. Section 5 is devoted to a summary and discussions. In appendix A we derive a formula, which is used to calculate the internal energy. In appendix B we present the form of Green's functions used to evaluate them efficiently in actual Monte Carlo simulation.

2. The models

The models we study in this paper are defined by the action

$$S = \frac{1}{g^2}\int_0^\beta dt\, \mathrm{tr}\left\{\frac{1}{2}(D_t X_i)^2 + \frac{1}{2}\psi_\alpha D_t \psi_\alpha - \frac{1}{4}[X_i, X_j]^2 - \frac{1}{2}\psi_\alpha(\gamma_i)_{\alpha\beta}[X_i, \psi_\beta]\right\}, \qquad (2.1)$$

where $D_t \equiv \partial_t - i[A(t), \,\cdot\,]$ represents the covariant derivative. The bosonic matrices $A(t)$, $X_i(t)$ $(i = 1, 2, \ldots, d)$ and the fermionic matrices $\psi_\alpha(t)$ $(\alpha = 1, 2, \ldots, p)$ are $N \times N$

Hermitian matrices, where $p = 4, 8, 16$ for $d = 3, 5, 9$, respectively. The models can be obtained formally by dimensionally reducing $\mathcal{N} = 1$ super Yang-Mills theory in $D = d + 1$ dimensions to one dimension, and they can be viewed as a 1d gauge theory, where $A(t)$, $X_i(t)$ and $\psi_\alpha(t)$ are the gauge field, adjoint scalars and spinors, respectively. The $p \times p$ symmetric matrices γ_i obey the Euclidean Clifford algebra $\{\gamma_i, \gamma_j\} = 2\delta_{ij}$. We impose periodic and anti-periodic boundary conditions on the bosonic and fermionic matrices, respectively. The extent β in the Euclidean time direction t then corresponds to the inverse temperature $\beta = T^{-1}$.

The action is invariant under the shifts

$$A(t) \mapsto A(t) + \alpha(t)\mathbf{1}\,, \quad X_i(t) \mapsto X_i(t) + x_i\mathbf{1}\,, \tag{2.2}$$

where $\alpha(t)$ is an arbitrary periodic function and x_i is an arbitrary constant. In order to remove the corresponding decoupled modes, we impose the conditions

$$\mathrm{tr}\, A(t) = 0\,, \quad \int_0^\beta dt\, \mathrm{tr}\, X_i(t) = 0 \quad (i = 1, 2, \ldots, d)\ . \tag{2.3}$$

The 't Hooft large-N limit corresponds to sending N to ∞ with the 't Hooft coupling constant $\lambda \equiv g^2 N$ fixed. Since the coupling constant g can be absorbed by rescaling the matrices and t appropriately, we can set λ to unity without loss of generality. This implies that we replace the prefactor $\frac{1}{g^2}$ in the action (2.1) by N in what follows.

We define the extent of the eigenvalue distribution and the Polyakov line as

$$R^2 \equiv \frac{1}{N\beta}\int_0^\beta dt\, \mathrm{tr}\left(X_i(t)\right)^2\,, \tag{2.4}$$

$$P \equiv \frac{1}{N}\mathrm{tr}\,\mathcal{P}\exp\left(i\int_0^\beta dt\, A(t)\right)\,, \tag{2.5}$$

where the symbol "$\mathcal{P}\exp$" represents the path-ordered exponential.

As a fundamental quantity in thermodynamics, the free energy $\mathcal{F} = -\frac{1}{\beta}\ln Z(\beta)$ is defined in terms of the partition function given in the present model as

$$Z(\beta) = \int [\mathcal{D}A]_\beta [\mathcal{D}X]_\beta [\mathcal{D}\psi]_\beta\, \mathrm{e}^{-S(\beta)}\,, \tag{2.6}$$

where the suffix of the measure $[\,\cdot\,]_\beta$ represents the period of the field to be path-integrated. However, the evaluation of the partition function $Z(\beta)$ is not straightforward in Monte Carlo simulation, which we use for the integration over the zero modes. We therefore study the internal energy defined by

$$E \equiv \frac{d}{d\beta}(\beta\mathcal{F}) = -\frac{d}{d\beta}\log Z(\beta)\,, \tag{2.7}$$

which has equivalent information as the free energy, given the boundary condition $\mathcal{F} = E$ at $T = 0$. Note also that the internal energy at $T = 0$ provides the ground state energy of the quantum mechanical system, which should vanish unless the supersymmetry is

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spontaneously broken. In appendix A we show that the internal energy E can be expressed as

$$\frac{E}{N^2} = \langle \mathcal{E}_{\rm b} \rangle + \langle \mathcal{E}_{\rm f} \rangle \,, \tag{2.8}$$

where the operators $\mathcal{E}_{\rm b}$ and $\mathcal{E}_{\rm f}$ are defined by

$$\mathcal{E}_{\rm b} \equiv -\frac{3}{4}\frac{1}{N\beta}\int_0^\beta dt \,\mathrm{tr}\left([X_i, X_j]^2\right), \tag{2.9}$$

$$\mathcal{E}_{\rm f} \equiv -\frac{3}{4}\frac{1}{N\beta}\int_0^\beta dt \,\mathrm{tr}\left(\psi_\alpha(\gamma_i)_{\alpha\beta}[X_i, \psi_\beta]\right) . \tag{2.10}$$

The symbol $\langle \,\cdot\, \rangle$ represents the expectation value with respect to the model (2.1).

Let us take the static gauge $\partial_t A(t) = 0$. Correspondingly we add the ghost term

$$S_{\rm gh} = N \int_0^\beta dt \,\mathrm{tr}\left(\partial_t \bar{c}(t) D_t c(t)\right) \tag{2.11}$$

to the action, where c, $\bar{c}$ are $N \times N$ matrices representing the ghosts. We make a Fourier expansion of the fields as

$$X_i(t) = \sum_n X_n^i \exp(in\omega t) \,, \qquad \psi_\alpha(t) = \sum_r \psi_r^\alpha \exp(ir\omega t) \,, \tag{2.12}$$

$$c(t) = \sum_{n\neq 0} c_n \exp(in\omega t) \,, \qquad \bar{c}(t) = \sum_{n\neq 0} \bar{c}_n \exp(-in\omega t) \,, \tag{2.13}$$

where $\omega = \frac{2\pi}{\beta}$ represents the unit of Matsubara frequencies, and the indices n and r take integers and half-integers, respectively, due to the imposed boundary conditions. In terms of the Fourier modes, the gauge-fixed action is written as

$$\tilde{S} = S_0 + S_{\rm kin} + S_{\rm int} \,, \tag{2.14}$$

$$S_0 \equiv -N\beta \,\mathrm{tr}\left\{\frac{1}{2}\left([A, X_0^i]\right)^2 + \frac{1}{4}\left([X_0^i, X_0^j]\right)^2\right\}, \tag{2.15}$$

$$S_{\rm kin} \equiv N\beta \,\mathrm{tr}\left\{\frac{1}{2}\sum_{n\neq 0}(n\omega)^2 X_{-n}^i X_n^i + \sum_{n\neq 0}(n\omega)^2 \bar{c}_n c_n + \frac{1}{2}\sum_r ir\omega \psi_{-r}\psi_r\right\}, \tag{2.16}$$

$$\begin{aligned} S_{\rm int} \equiv -N\beta \,\mathrm{tr}\Bigg\{ &\sum_{n\neq 0} n\omega X_{-n}^i [A, X_n^i] + \sum_{n\neq 0} n\omega \bar{c}_n [A, c_n] + \frac{i}{2}\sum_r \psi_{-r}[A, \psi_r] \\ &+\frac{1}{2}\sum_{r,s}\psi_r \gamma_i [X_{-r-s}^i, \psi_s] + \frac{1}{2}\sum_{n\neq 0}[A, X_{-n}^i][A, X_n^i] + \frac{1}{4}{\sum_{npq}}' [X_{-n-p-q}^i, X_n^j][X_p^i, X_q^j]\Bigg\}, \end{aligned} \tag{2.17}$$

where the symbol $\sum'$ implies that the $m{=}p{=}q{=}0$ term is excluded.

3. Leading order calculation

In this section we consider the high temperature limit [22, 10, 23], which corresponds to the leading order calculation at high T. From (2.14), one can easily see that all the non-zero modes decouple, and one is left with the zero modes governed by the action (2.15). By rescaling the zero modes as

$$\tilde{A}_i \equiv T^{-1/4} X_0^i \quad (i = 1, 2, \ldots, d) \, , \quad \tilde{A}_D \equiv T^{-1/4} A \, , \tag{3.1}$$

where we recall that $D \equiv d + 1$, the zero-mode action can be brought into the form

$$S_0 = \frac{1}{4} N \mathrm{tr} \, (\tilde{F}_{\mu\nu})^2 \, , \quad \tilde{F}_{\mu\nu} = -i \, [\tilde{A}_\mu, \tilde{A}_\nu] \, . \tag{3.2}$$

Here and henceforth, the Greek indices μ, ν are assumed to run over $1, 2, \ldots, D$. The dimensionally reduced (DR) model (3.2) is nothing but the bosonic part of the IKKT type matrix model [24 − 26]. The leading behavior of the observables at high temperature can be obtained as

$$\langle R^2 \rangle \simeq T^{1/2} \left\langle \frac{1}{N} \mathrm{tr} \, (\tilde{A}_i)^2 \right\rangle_{\mathrm{DR}} = \chi_1 \, T^{1/2} \, , \tag{3.3}$$

$$\langle P \rangle \simeq 1 - \frac{1}{2} T^{-3/2} \left\langle \frac{1}{N} \mathrm{tr} \, (\tilde{A}_D)^2 \right\rangle_{\mathrm{DR}} = 1 - \frac{1}{2d} \chi_1 \, T^{-3/2} \, , \tag{3.4}$$

$$\frac{1}{N^2} E \simeq \frac{3}{4} T \left\langle \frac{1}{N} \mathrm{tr} \, (\tilde{F}_{ij})^2 \right\rangle_{\mathrm{DR}} = \frac{3}{4} \chi_2 \, T \, , \tag{3.5}$$

where $\langle \, \cdot \, \rangle_{\mathrm{DR}}$ represents the expectation value with respect to the DR model, and the coefficients χ_1 and χ_2 are given as

$$\chi_1 \equiv \left\langle \frac{1}{N} \, \mathrm{tr} \, (\tilde{A}_i)^2 \right\rangle_{\mathrm{DR}} = \frac{d}{D} \left\langle \frac{1}{N} \, \mathrm{tr} \, (\tilde{A}_\mu)^2 \right\rangle_{\mathrm{DR}} \, , \tag{3.6}$$

$$\chi_2 \equiv \left\langle \frac{1}{N} \, \mathrm{tr} \, (\tilde{F}_{ij})^2 \right\rangle_{\mathrm{DR}} = \frac{{}_d\mathrm{C}_2}{{}_D\mathrm{C}_2} \left\langle \frac{1}{N} \, \mathrm{tr} \, (\tilde{F}_{\mu\nu})^2 \right\rangle_{\mathrm{DR}} = (d-1) \left(1 - \frac{1}{N^2} \right) \, . \tag{3.7}$$

We note that the expectation values appearing here are standard quantities calculated in the DR model (3.2) by various methods [25, 26]. In particular, the quantity in eq. (3.7) can be obtained exactly by simply rescaling the dynamical variables or by writing down the Schwinger-Dyson equation [25].

4. Next-leading order calculation

As is clear from the previous section, the leading order results are insensitive to the existence of fermions. In order to see their effects, we need to proceed to the next-leading order calculation, which involves the integration over the non-zero modes.

For that purpose, let us rescale the non-zero modes as

$$\tilde{X}_n^i = \beta^{-1/2} \, X_n^i \, , \quad \tilde{\psi}_r = \psi_r \, , \quad \tilde{c}_n = \beta^{-1/2} \, \tilde{c}_n \, , \quad \tilde{\bar{c}}_n = \beta^{-1/2} \, \tilde{\bar{c}}_n \, , \tag{4.1}$$

where $n \neq 0$, so that the kinetic terms take the canonical form

$$S_{\mathrm{kin}} \equiv N \; \mathrm{tr} \left\{ \frac{1}{2} \sum_{n \neq 0} (2\pi n)^2 \, \tilde{X}_{-n}^i \tilde{X}_n^i + \sum_{n \neq 0} (2\pi n)^2 \, \tilde{\bar{c}}_n \tilde{c}_n + \frac{1}{2} \sum_r 2\pi i r \, \tilde{\psi}_{-r} \tilde{\psi}_r \right\} \, . \tag{4.2}$$

Then the propagators are given by

$$\left\langle\!\!\left\langle (\tilde{X}^i_m)_{ab}(\tilde{X}^j_n)_{cd} \right\rangle\!\!\right\rangle = \frac{1}{(2\pi n)^2 N}\ \delta_{ij}\delta_{m,-n}\delta_{ad}\delta_{bc}\,, \tag{4.3}$$

$$\left\langle\!\!\left\langle (\tilde{\psi}_{\alpha r})_{ab}(\tilde{\psi}_{\beta s})_{cd} \right\rangle\!\!\right\rangle = \frac{1}{2\pi i r N}\ \delta_{\alpha\beta}\delta_{r,-s}\delta_{ad}\delta_{bc}\,, \tag{4.4}$$

$$\left\langle\!\!\left\langle (\tilde{c}_m)_{ab}(\tilde{\bar{c}}_n)_{cd} \right\rangle\!\!\right\rangle = \frac{1}{(2\pi n)^2 N}\ \delta_{mn}\delta_{ad}\delta_{bc}\,, \tag{4.5}$$

where the symbol $\langle\!\langle\ \cdot\ \rangle\!\rangle$ represents integrating *only the non-zero modes* using the quadratic terms (4.2). The interaction terms are given by $S_{\rm int} \equiv -\sum_{i=1}^{6}\mathcal{V}_i$, where

$$\begin{aligned}
\mathcal{V}_1 &\equiv \sqrt{\epsilon}N\sum_{n\neq 0} n\,\mathrm{tr}\left(\tilde{X}^i_{-n}[\tilde{A}_D,\tilde{X}^i_n]\right), \quad \mathcal{V}_2 \equiv \sqrt{\epsilon}N\sum_{n\neq 0} n\,\mathrm{tr}\left(\tilde{\bar{c}}_n[\tilde{A}_D,\tilde{c}_n]\right), \\
\mathcal{V}_3 &\equiv \frac{i}{2}\sqrt{\epsilon}N\sum_{r}\mathrm{tr}\left(\tilde{\psi}_{-r}[\tilde{A}_D,\tilde{\psi}_r]\right), \quad \mathcal{V}_4 \equiv \frac{1}{2}\sqrt{\epsilon}N\sum_{r}\mathrm{tr}\left(\tilde{\psi}_{-r}\gamma_i[\tilde{A}_i,\tilde{\psi}_r]\right), \\
\mathcal{V}_5 &\equiv \frac{1}{2}\epsilon N\sum_{n\neq 0}\mathrm{tr}\left([\tilde{A}_D,\tilde{X}^i_{-n}][\tilde{A}_D,\tilde{X}^i_n]\right), \\
\mathcal{V}_6 &\equiv \frac{1}{2}\epsilon N\sum_{n\neq 0}\mathrm{tr}\left([\tilde{A}_i,\tilde{X}^j_{-n}][\tilde{A}_i,\tilde{X}^j_n]+[\tilde{A}_i,\tilde{X}^j_{-n}][\tilde{X}^i_n,\tilde{A}_j]\right).
\end{aligned} \tag{4.6}$$

We have introduced the expansion parameter[2] $\epsilon = \beta^{3/2}$, and omitted terms, which are irrelevant to the calculations at the next-leading order.

First we calculate the extent of the eigenvalue distribution (2.4), which can be decomposed as

$$R^2 = \frac{1}{N}\mathrm{tr}\,(X^i_0)^2 + \frac{1}{N}\sum_{n\neq 0}\mathrm{tr}\,(X^i_n X^i_{-n})\,. \tag{4.7}$$

Let us consider the first term. The leading order contribution is given by (3.3). At the next-leading order, we use the vertices (4.6) and integrate over the non-zero modes at one-loop making use of the formulae

$$\sum_{n\neq 0}\frac{1}{(2\pi n)^2} = \frac{1}{12}\,, \quad \sum_{r}\frac{1}{(2\pi r)^2} = \frac{1}{4} \tag{4.8}$$

to sum over the Matsubara frequencies in the loop. This gives rise to the operators written in terms of zero modes as

$$\begin{aligned}
\mathcal{O}_1 &\equiv \frac{1}{2}\langle\!\langle\,(\mathcal{V}_1)^2\,\rangle\!\rangle = \frac{d}{6}N\beta^{3/2}\mathrm{tr}\,(\tilde{A}_D)^2\,, \qquad \mathcal{O}_2 \equiv \frac{1}{2}\langle\!\langle\,(\mathcal{V}_2)^2\,\rangle\!\rangle = -\frac{1}{12}N\beta^{3/2}\mathrm{tr}\,(\tilde{A}_D)^2\,, \\
\mathcal{O}_3 &\equiv \frac{1}{2}\langle\!\langle\,(\mathcal{V}_3)^2\,\rangle\!\rangle = -\frac{p}{8}N\beta^{3/2}\mathrm{tr}\,(\tilde{A}_D)^2\,, \quad \mathcal{O}_4 \equiv \frac{1}{2}\langle\!\langle\,(\mathcal{V}_4)^2\,\rangle\!\rangle = \frac{p}{8}N\beta^{3/2}\mathrm{tr}\,(\tilde{A}_i)^2\,,
\end{aligned}$$

[2] If we left the 't Hooft coupling constant λ arbitrary, we would find that the expansion parameter is given by $\epsilon\sqrt{\lambda} = \sqrt{\lambda/T^3}$. This is what one might have deduced on dimensional grounds, since the 't Hooft coupling constant has the dimension of $(\mathrm{mass})^3$ in the present models.

$$\mathcal{O}_5 \equiv \langle\!\langle \mathcal{V}_5 \rangle\!\rangle = -\frac{d}{12} N\beta^{3/2} \mathrm{tr}\,(\tilde{A}_D)^2 \,, \qquad \mathcal{O}_6 \equiv \langle\!\langle \mathcal{V}_6 \rangle\!\rangle = -\frac{d-1}{12} N\beta^{3/2} \mathrm{tr}\,(\tilde{A}_i)^2 \,. \tag{4.9}$$

Summing up these operators, we obtain

$$\mathcal{O} = \sum_{j=1}^{6} \mathcal{O}_j = -\left(\frac{d-1}{12} - \frac{p}{8}\right) N\beta^{3/2} \left\{ \mathrm{tr}\,(\tilde{A}_i)^2 - \mathrm{tr}\,(\tilde{A}_D)^2 \right\} \,. \tag{4.10}$$

Using this operator, we can evaluate the first term in eq. (4.7) as

$$\begin{aligned} \frac{1}{N}\left\langle \mathrm{tr}\,(X_0^i)^2 \right\rangle &\simeq \left\langle \frac{1}{N}\, \mathrm{tr}\,(X_0^i)^2 \right\rangle_{\mathrm{DR}} + \left\langle \frac{1}{N}\, \mathrm{tr}\,(X_0^i)^2 \cdot \mathcal{O} \right\rangle_{\mathrm{DR,c}} \\ &= \chi_1\, T^{1/2} - \left(\frac{d-1}{12} - \frac{p}{8}\right)(\chi_3 - \chi_4)\, T^{-1} + \mathrm{O}(T^{-5/2}) \,, \end{aligned} \tag{4.11}$$

where the subscript "c" implies that the connected part is taken, and we define the coefficients χ_3 and χ_4 by

$$\chi_3 \equiv \left\langle \mathrm{tr}\,(\tilde{A}_i)^2 \cdot \mathrm{tr}\,(\tilde{A}_j)^2 \right\rangle_{\mathrm{DR,c}} \,, \quad \chi_4 \equiv \left\langle \mathrm{tr}\,(\tilde{A}_i)^2 \cdot \mathrm{tr}\,(\tilde{A}_D)^2 \right\rangle_{\mathrm{DR,c}} \,. \tag{4.12}$$

The second term of eq. (4.7) can be calculated at the next-leading order using the propagator (4.3), and we get

$$\frac{1}{N} \sum_{n\neq 0} \left\langle \mathrm{tr}\,(X_n^i X_{-n}^i) \right\rangle \simeq \frac{1}{N} \sum_{n\neq 0} \left\langle\!\!\left\langle \mathrm{tr}\,(X_n^i X_{-n}^i) \right\rangle\!\!\right\rangle = \frac{d}{12}\, T^{-1} + \mathrm{O}(T^{-5/2}) \,. \tag{4.13}$$

Adding the two terms, we get the result at the next-leading order as

$$\langle R^2 \rangle = \chi_1 T^{1/2} + \left\{ \frac{d}{12} - \left(\frac{d-1}{12} - \frac{p}{8}\right)(\chi_3 - \chi_4) \right\} T^{-1} + \mathrm{O}(T^{-5/2}) \,. \tag{4.14}$$

Let us calculate the internal energy (2.7) using (2.8). The operators $\mathcal{E}_{\mathrm{b}}$ and $\mathcal{E}_{\mathrm{f}}$ can be decomposed as

$$\mathcal{E}_{\mathrm{b}} = \frac{3T}{4N} \mathrm{tr}\,(\tilde{F}_{ij})^2 - \frac{3}{N^2\beta} \mathcal{V}_6 + \cdots \,, \quad \mathcal{E}_{\mathrm{f}} = -\frac{3}{2N^2\beta} \mathcal{V}_4 + \cdots \,, \tag{4.15}$$

where we have omitted terms irrelevant at the next-leading order. The expectation values can be calculated as

$$\begin{aligned} \langle \mathcal{E}_{\mathrm{b}} \rangle &\simeq \frac{3}{4} T \left\{ \left\langle \frac{1}{N}\, \mathrm{tr}\,(\tilde{F}_{ij})^2 \right\rangle_{\mathrm{DR}} + \left\langle \frac{1}{N}\, \mathrm{tr}\,(\tilde{F}_{ij})^2 \cdot \mathcal{O} \right\rangle_{\mathrm{DR,c}} \right\} - \frac{3}{N^2\beta} \left\langle \langle\!\langle \mathcal{V}_6 \rangle\!\rangle \right\rangle_{\mathrm{DR}} \\ &= \frac{3}{4}\chi_2\, T + \left\{ \frac{1}{4}(d-1)\chi_1 - \frac{3}{4}\left(\frac{d-1}{12} - \frac{p}{8}\right)(\chi_5 - \chi_6) \right\} T^{-1/2} + \mathrm{O}\big(T^{-2}\big) \,, \end{aligned} \tag{4.16}$$

$$\langle \mathcal{E}_{\mathrm{f}} \rangle \simeq -\frac{3}{2N^2\beta} \left\langle \langle\!\langle (\mathcal{V}_4)^2 \rangle\!\rangle \right\rangle_{\mathrm{DR}} = -\frac{3p}{8}\, \chi_1\, T^{-1/2} + \mathrm{O}\big(T^{-2}\big) \,, \tag{4.17}$$

where we define the coefficients

$$\chi_5 \equiv \left\langle \mathrm{tr}\,(\tilde{F}_{ij})^2 \cdot \mathrm{tr}\,(\tilde{A}_k)^2 \right\rangle_{\mathrm{DR,c}} \,, \quad \chi_6 \equiv \left\langle \mathrm{tr}\,(\tilde{F}_{ij})^2 \cdot \mathrm{tr}\,(\tilde{A}_D)^2 \right\rangle_{\mathrm{DR,c}} \,. \tag{4.18}$$

Adding these terms, we get the result for the internal energy as

$$\frac{E}{N^2} = \frac{3}{4}\chi_2 T - \frac{3}{4}\left(\frac{d-1}{12} - \frac{p}{8}\right)(\chi_5 - \chi_6 - 4\chi_1)\, T^{-1/2} + \mathrm{O}\big(T^{-2}\big) \ . \tag{4.19}$$

Similarly the Polyakov line can be calculated as

$$\begin{aligned} \langle P \rangle \simeq\ & 1 - \frac{1}{2} T^{-3/2} \left\{ \left\langle \frac{1}{N} \mathrm{tr}\,(\tilde{A}_D)^2 \right\rangle_{\mathrm{DR}} + \left\langle \frac{1}{N} \mathrm{tr}\,(\tilde{A}_D)^2 \cdot \mathcal{O} \right\rangle_{\mathrm{DR,c}} \right\} \\ & + \frac{1}{24} T^{-3} \left\langle \frac{1}{N} \mathrm{tr}\,(\tilde{A}_D)^4 \right\rangle_{\mathrm{DR}} \\ =\ & 1 - \frac{1}{2d}\chi_1 T^{-3/2} + \left\{ \frac{1}{24}\chi_8 + \frac{1}{2}\left(\frac{d-1}{12} - \frac{p}{8}\right)(\chi_4 - \chi_7) \right\} T^{-3} + \mathrm{O}\big(T^{-9/2}\big) \, , \end{aligned} \tag{4.20}$$

where we define the coefficients χ_7 and χ_8 by

$$\chi_7 \equiv \left\langle \mathrm{tr}\,(\tilde{A}_D)^2 \cdot \mathrm{tr}\,(\tilde{A}_D)^2 \right\rangle_{\mathrm{DR,c}} \, , \quad \chi_8 \equiv \left\langle \frac{1}{N} \mathrm{tr}\,(\tilde{A}_D)^4 \right\rangle_{\mathrm{DR}} \, . \tag{4.21}$$

Thus we have obtained various quantities up to the next-leading order with the coefficients χ_i, which can be obtained by Monte Carlo evaluation of the connected Green's functions in the DR model (3.2). In practice, we rewrite the Green's functions as described in appendix B in order to increase the statistics. The values of χ_i obtained in this way[3] for $d = 3, 5, 9$ and for various N are summarized in table 1. In order to see the large-N behavior [25], we plot the values of χ_i against $1/N^2$ for $12 \le N \le 32$ in figure 1. We observe that the data points for $N = 16, 20, 32$ lie on a straight line. This enables us to obtain the large-N extrapolated values shown in table 1.

Using the coefficients χ_i extrapolated to $N = \infty$, we evaluate the expressions (4.14), (4.19) and (4.20). The results for the bosonic case can be readily obtained by setting $p = 0$ in the same expressions. In figure 2 we show various quantities as a function of T for $d = 3$ (left column) and $d = 9$ (right column), respectively. The curves represent the results of the high temperature expansion.[4] The solid lines represent the leading order results, which are the same for the bosonic and supersymmetric cases. The dashed lines and the dash-dotted lines represent the next-leading order results for the bosonic case and the supersymmetric case, respectively. For comparison, we also plot the recent Monte Carlo data obtained at finite T for the bosonic model with $d = 3$ [23] and $d = 9$ [17], and for the supersymmetric model with $d = 9$ [15]. In both bosonic and supersymmetric cases, the high temperature expansion including the next-leading order terms seems to be valid at $T \gtrsim 2$.

[3]The heat-bath algorithm as described in ref. [25] has been used for Monte Carlo simulation. We have made 25M sweeps for $d = 3, 5$ and 8M sweeps for $d = 9$ to obtain the data. We have checked that the expectation value appearing in the definition (3.6) of χ_1 agrees with the previous results [25].

[4]In refs. [23, 17, 15], we have presented the results of the high temperature expansion using the coefficients χ_i obtained at the same N as those used for Monte Carlo simulations at finite temperature. The quality of the agreement with the Monte Carlo data at high T is almost the same as in the present plots.

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d	N	χ_1	χ_3	χ_4	χ_5	χ_6	χ_7	χ_8
3	4	1.48(1)	2.79(8)	-0.48(5)	3.29(1)	-0.70(4)	1.40(5)	0.60(1)
3	8	1.6579(4)	2.756(7)	-0.751(4)	3.294(6)	-1.162(5)	1.419(4)	0.6581(4)
3	10	1.6305(3)	2.56(1)	-0.718(9)	3.32(1)	-1.10(1)	1.33(1)	0.6342(9)
3	12	1.6295(3)	2.319(6)	-0.708(4)	3.195(7)	-1.140(6)	1.245(4)	0.6239(3)
3	16	1.6229(1)	2.230(4)	-0.720(3)	3.148(5)	-1.102(5)	1.223(3)	0.6156(1)
3	20	1.6198(1)	2.070(4)	-0.677(9)	2.970(1)	-1.04(1)	1.142(7)	0.6123(1)
3	32	1.61697(4)	1.940(6)	-0.633(3)	2.797(9)	-0.981(5)	1.069(4)	0.60780(5)
3	∞	1.6150(1)	1.83(1)	-0.6039(6)	2.676(9)	-0.940(2)	1.016(5)	0.6051(2)
5	4	1.821(1)	1.56(1)	-0.209(3)	3.53(1)	-0.732(8)	0.481(4)	0.2812(8)
5	8	1.8331(2)	1.181(2)	-0.2157(7)	3.564(7)	-0.703(2)	0.4087(8)	0.2778(1)
5	10	1.8356(6)	1.179(6)	-0.232(2)	3.68(1)	-0.732(8)	0.421(2)	0.2785(2)
5	12	1.8377(3)	1.153(3)	-0.229(1)	3.633(9)	-0.703(2)	0.414(1)	0.2788(1)
5	16	1.83935(9)	1.141(2)	-0.2251(9)	3.658(7)	-0.723(2)	0.4084(8)	0.27893(3)
5	20	1.8387(1)	1.104(2)	-0.2220(8)	3.544(6)	-0.718(2)	0.3985(8)	0.27866(4)
5	32	1.8393(3)	1.041(3)	-0.2282(5)	3.35(1)	-0.744(1)	0.3909(7)	0.27874(1)
5	∞	1.8382(8)	1.01(1)	-0.229(3)	3.27(4)	-0.751(9)	0.384(1)	0.27868(5)
9	4	2.191(1)	0.769(5)	-0.0925(5)	3.99(1)	-0.558(2)	0.1681(6)	0.1199(1)
9	8	2.2700(2)	0.746(1)	-0.0861(3)	4.34(1)	-0.510(1)	0.1595(2)	0.12894(3)
9	10	2.2810(5)	0.766(3)	-0.0859(6)	4.44(2)	-0.506(3)	0.1615(5)	0.13045(7)
9	12	2.2854(3)	0.751(4)	-0.0863(6)	4.44(2)	-0.510(1)	0.1602(2)	0.13114(3)
9	16	2.2901(1)	0.746(2)	-0.0886(5)	4.43(1)	-0.525(3)	0.1617(2)	0.13163(2)
9	20	2.2932(3)	0.734(3)	-0.0912(9)	4.40(2)	-0.55(1)	0.1631(6)	0.13204(3)
9	32	2.29566(7)	0.730(6)	-0.082(1)	4.38(2)	-0.59(1)	0.1399(1)	0.13234(1)
9	∞	2.2975(1)	0.719(6)	-0.082(6)	4.36(2)	-0.61(2)	0.14(2)	0.13257(5)

Table 1: The values of χ_i $(i = 1, 3, \ldots, 8)$ for various d and N obtained by Monte Carlo simulation of the corresponding DR model. The values at $N = \infty$ are obtained by extrapolating the results for $N = 16, 20, 32$ as shown in figure 1.

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5. Summary and discussions

In this paper we have formulated the high temperature expansion for the supersymmetric matrix quantum mechanics with 4, 8 and 16 supercharges. While the non-zero modes become weakly coupled at high temperature, the zero modes remain strongly coupled and hence they have to be treated non-perturbatively. This makes the problem nontrivial, but we are able to obtain the next-leading order terms by evaluating connected Green's function in the bosonic IKKT model using Monte Carlo simulation. Since the fermions decouple at the leading order, it is highly motivated to carry out the next-leading order calculation. Indeed, our results including the next-leading order terms are in good agreement with the finite temperature calculations down to $T \simeq 2$ in units of the 't Hooft coupling constant. Note also that Monte Carlo evaluation of the connected Green's functions in the bosonic

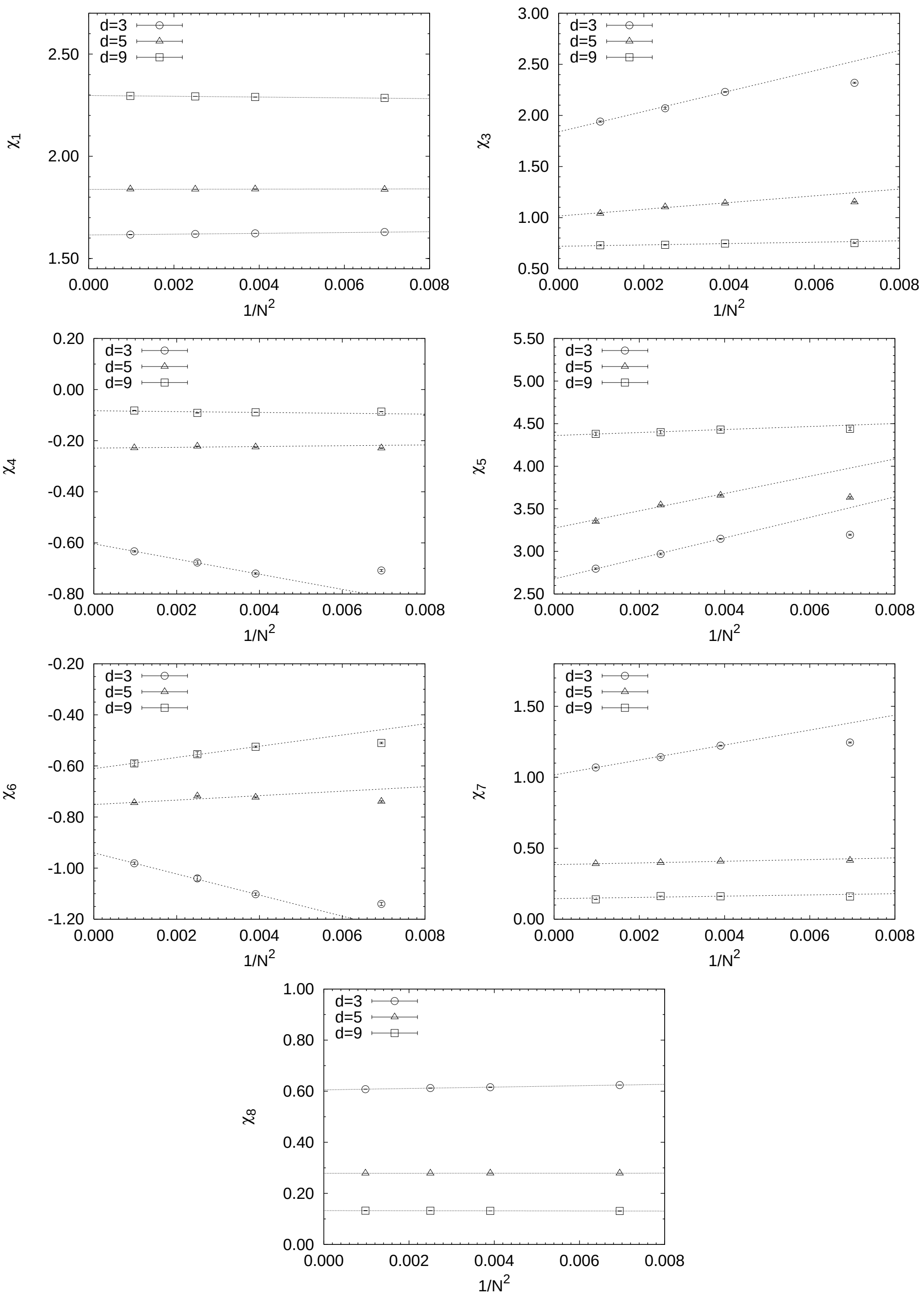

Figure 1: The coefficients χ_i $(i = 1, 3, \ldots, 8)$ for $d = 3, 5, 9$ and $12 \leq N \leq 32$ evaluated by Monte Carlo simulation of the corresponding DR model are plotted against $1/N^2$. The straight lines represent fits to the expected large-N behavior $a + b/N^2$ using the $N = 16, 20, 32$ data points. The extrapolated values are shown in table 1 as results at $N = \infty$.

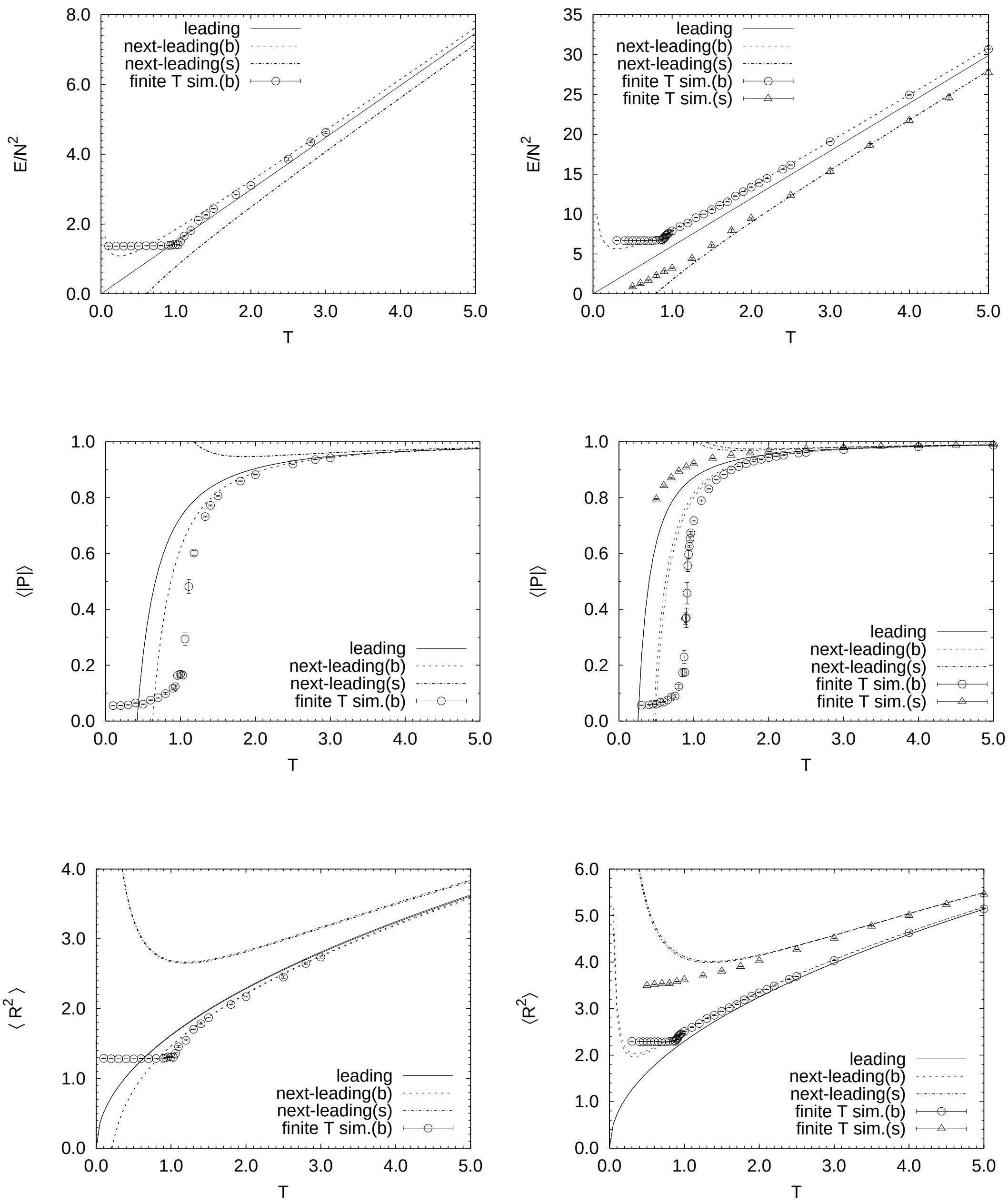

Figure 2: Various quantities are shown for $d = 3$ (left column) and $d = 9$ (right column). The curves represent the results obtained by the high temperature expansion using the large-N extrapolated values for χ_i shown in table 1. The solid lines represent the leading order results, which are the same for the bosonic and supersymmetric cases. The dashed lines and the dash-dotted lines represent the next-leading order results for the bosonic case and the supersymmetric case, respectively. The circles and squares represent the results obtained by Monte Carlo simulation at finite T for the bosonic model with $N = 16$ [23, 17] and for the supersymmetric model with $N = 12$ [15], respectively.

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IKKT model is by far easier than simulating the supersymmetric matrix quantum mechanics at finite temperature directly. This enables us to study the behavior at larger N and to make a reliable large-N extrapolation. Our results confirm that the values of N used in finite temperature simulations are already large enough to probe the 't Hooft large-N limit at high temperature.

It is straightforward to extend our calculation to higher orders. For that, one needs to evaluate connected Green's functions with more than two operators inserted. That will require more statistics in Monte Carlo evaluation. For finite N, it is anticipated that the connected Green's functions with many insertions of the $\mathrm{tr}\,(\tilde{A}_\mu)^2$ operator would be divergent [27], and the order at which such divergence shows up would grow linearly with N. This property of the high temperature expansion is reminiscent of the infrared instability observed in Monte Carlo simulation of the supersymmetric model at finite temperature [15]. We note, however, that the divergence in the high temperature expansion occurs also in the bosonic case, in which the finite temperature calculations exhibit no such instability.

It is worth while to generalize our formulation to higher dimensions. For instance, an interesting phase structure is expected in 2d U(N) $\mathcal{N} = 8$ super Yang-Mills theory on a finite torus.[5] In the strong coupling and low temperature regime, the gauge/gravity duality predicts [10, 31] that there exists a phase transition corresponding to the black-string/black-hole transition [32] in the dual gravity theory. In the weak coupling and high temperature regime, on the other hand, one can study the theory by dimensionally reduced 1d bosonic model. In ref. [10], the phase transition observed in the latter regime has been conjectured to be a continuation of the phase transition in the former regime. In ref. [17] the critical region of the dimensionally reduced 1d bosonic model has been studied more carefully, and a new phase characterized by the non-uniform eigenvalue distribution of the holonomy matrix has been discovered. Of particular interest from the viewpoint of the gauge/gravity duality is to investigate the fate of this new phase as one lowers the temperature. Calculations including the next-leading order terms in the high temperature expansion would be useful for such purposes.

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A. Derivation of the formula for the internal energy

In this appendix we derive the formula (2.8), which is used to calculate the internal energy by the high temperature expansion. The case without fermions is given in ref. [17]. Let us

[5] The situation becomes more involved in dimensions higher than two. Recently cascade transitions from the black p-brane solution to the black $(p-1)$-brane solution have been found in the dual gravity theories [28]. On the other hand, the high temperature limit of the 4d U(N) $\mathcal{N} = 4$ super Yang-Mills theory on a finite torus, for instance, is described by the dimensionally reduced 3d bosonic model. Analogous cascade transitions were observed earlier in the large-N pure Yang-Mills theory on a 3d torus [29] and in a related model [30].

first rewrite (2.7) as

$$E = -\frac{1}{Z(\beta)} \lim_{\Delta\beta \to 0} \frac{Z(\beta') - Z(\beta)}{\Delta\beta} , \tag{A.1}$$

where $\beta' = \beta + \Delta\beta$, and represent $Z(\beta')$ for later convenience as

$$Z(\beta') = \int [\mathcal{D}A']_{\beta'} [\mathcal{D}X']_{\beta'} [\mathcal{D}\psi']_{\beta'} \, \mathrm{e}^{-S'} , \tag{A.2}$$

where S' is obtained from S given in (2.1) by replacing β, t, $A(t)$, $X_i(t)$, $\psi_\alpha(t)$ with β', t', $A'(t')$, $X_i'(t')$, $\psi_\alpha'(t')$. In order to relate $Z(\beta')$ to $Z(\beta)$, we consider the transformation

$$t' = \frac{\beta'}{\beta} t , \quad A'(t') = \frac{\beta}{\beta'} A(t) , \quad X_i'(t') = \sqrt{\frac{\beta'}{\beta}} X_i(t) , \quad \psi'(t') = \psi(t) , \tag{A.3}$$

where the constant factors are motivated on dimensional grounds, and we have $[\mathcal{D}X']_{\beta'} = [\mathcal{D}X]_\beta$, $[\mathcal{D}\psi']_{\beta'} = [\mathcal{D}\psi]_\beta$ and $[\mathcal{D}A']_{\beta'} = [\mathcal{D}A]_\beta$. Under this transformation, the kinetic term in S' reduces to that in S, but the interaction term transforms non-trivially as

$$\int_0^{\beta'} dt' \, \mathrm{tr} \Big([X_i'(t'), X_j'(t')] \Big)^2 = \left(\frac{\beta'}{\beta} \right)^3 \int_0^\beta dt \, \mathrm{tr} \Big([X_i(t), X_j(t)] \Big)^2 , \tag{A.4}$$

$$\int_0^{\beta'} dt \, \mathrm{tr} \Big(\psi_\alpha(t') [X_i{}'(t'), \psi_\beta'(t')] \Big) = \left(\frac{\beta'}{\beta} \right)^{3/2} \int_0^\beta dt \, \mathrm{tr} \Big(\psi_\alpha [X_i(t), \psi_\beta(t)] \Big) . \tag{A.5}$$

This gives us the relation

$$Z(\beta') = Z(\beta) \left(1 - N^2 \Delta\beta \left(\mathcal{E}_\mathrm{b} + \mathcal{E}_\mathrm{f} \right) + \mathrm{O}((\Delta\beta)^2) \right) , \tag{A.6}$$

where the coefficients $\mathcal{E}_\mathrm{b}$ and $\mathcal{E}_\mathrm{f}$ are defined by (2.9) and (2.10). Plugging these into (A.1), we get (2.8). Thus we are able to express the internal energy E in terms of the expectation values, which can be calculated directly by Monte Carlo simulation.

B. Increasing statistics by exploiting SO(D) symmetry

In eqs. (3.6) and (3.7), we have rewritten the expectation values that define χ_1 and χ_2 by exploiting the SO(D) symmetry of the DR model (3.2). Similar rewriting can be done also for the other coefficients χ_i ($i = 3, \ldots, 8$) defined in eqs. (4.12), (4.18) and (4.21) as presented below. In actual measurements in the Monte Carlo simulation, we can increase the statistics considerably by using these forms instead of the original ones.

$$\begin{aligned}
\chi_3 &= \frac{d}{D} \sum_\mu \left\langle \mathrm{tr}\,(\tilde{A}_\mu)^2 \cdot \mathrm{tr}\,(\tilde{A}_\mu)^2 \right\rangle_{\mathrm{DR,C}} + \frac{2\,{}_d\mathrm{C}_2}{{}_D\mathrm{C}_2} \sum_{\mu<\nu} \left\langle \mathrm{tr}\,(\tilde{A}_\mu)^2 \cdot \mathrm{tr}\,(\tilde{A}_\nu)^2 \right\rangle_{\mathrm{DR,C}} , \\
\chi_4 &= \frac{d}{{}_D\mathrm{C}_2} \sum_{\mu<\nu} \left\langle \mathrm{tr}\,(\tilde{A}_\mu)^2 \cdot \mathrm{tr}\,(\tilde{A}_\nu)^2 \right\rangle_{\mathrm{DR,C}} , \\
\chi_5 &= \frac{2\,{}_d\mathrm{C}_2}{{}_D\mathrm{C}_2} \sum_{\mu<\nu} \left\langle \mathrm{tr}\,(\tilde{F}_{\mu\nu})^2 \cdot \mathrm{tr} \left\{ (\tilde{A}_\mu)^2 + (\tilde{A}_\nu)^2 \right\} \right\rangle_{\mathrm{DR,C}}
\end{aligned}$$

$$+\frac{2(d-2)_d\mathrm{C}_2}{(D-2)_D\mathrm{C}_2}\sum_{\mu<\nu}\sum_{\lambda\neq\mu,\nu}\left\langle \mathrm{tr}\,(\tilde{F}_{\mu\nu})^2\cdot\mathrm{tr}\,(\tilde{A}_\lambda)^2\right\rangle_{\mathrm{DR,C}},$$

$$\chi_6 = \frac{2_d\mathrm{C}_2}{(D-2)_D\mathrm{C}_2}\sum_{\mu<\nu}\sum_{\lambda\neq\mu,\nu}\left\langle \mathrm{tr}\,(\tilde{F}_{\mu\nu})^2\cdot\mathrm{tr}\,(\tilde{A}_\lambda)^2\right\rangle_{\mathrm{DR,C}},$$

$$\chi_7 = \frac{1}{D}\sum_{\mu}\left\langle \mathrm{tr}\,(\tilde{A}_\mu)^2\cdot\mathrm{tr}\,(\tilde{A}_\mu)^2\right\rangle_{\mathrm{DR,C}},$$

$$\chi_8 = \frac{1}{D}\left\langle \frac{1}{N}\mathrm{tr}\,(\tilde{A}_\mu)^4\right\rangle_{\mathrm{DR}}. \tag{B.1}$$

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